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APPLICATION OF MULTIDIMENSIONAL HODRICK-PRESCOTT FILTER FOR CALCULATION AND PREDICTION OF POTENTIAL GRP AND SFO OUTPUT GAP

Аннотация: В статье показано, что переход от одномерного фильтра Ходрика-Прескотта, широко используемого Центральным банком России, к многомерному позволяет улучшить оценки потенциального выпуска и разрыва выпуска, а также сделать прогнозы этих показателей более точными. Использование дополнительной информации по таким показателям, как инфляция и безработица, частично решает основную проблему одномерного фильтра Ходрика-Прескотта и снижает чувствительность результатов к экстремальным наблюдениям за выходным индикатором. Кроме того, в ходе работы, после применения усовершенствованной методики, были построены прогнозы потенциального ВРП и разрыва в выпуске до января 2022 года и выявлена проблема, касающаяся статистических данных.

Ключевые слова: многомерный фильтр, HP, потенциальный выход, выходной разрыв, кривая Филлипса, закон Окена.

Annotation: The article shows that the transition from the one-dimensional Hodrick-Prescott filter, widely used by the Central Bank of Russia, to a multidimensional one makes it possible to improve estimates of potential output and output gap, as well as to make forecasts of these indicators more accurate. The use of additional information on indicators such as inflation and unemployment partly solves the main problem of the one-dimensional Hodrick-Prescott filter and reduces

the sensitivity of the results to extreme observations of the output indicator. In addition, in the course of work, after applying the improved method, forecasts of potential GRP and output gap were built until January 2022 and a problem concerning statistical data was identified.

Keywords: multidimensional filter, HP, potential output, output gap, Phillips curve, Oaken's law.

Introduction

In the decision-making process in the field of monetary policy, one of the key indicators is the size of the output gap, calculated based on estimates of potential output. The output gap is usually understood as the percentage deviation of the actual output from some estimated potential level.

Currently, there are many methods for estimating the "output gap", but there is no clear opinion among economists about the optimal way of estimating. One of the methods used by the Central Bank of Russia to estimate this indicator is the one–dimensional Hodrick-Prescott filter (HP). The trend obtained on the basis of the HP filter represents a potential release, and the cycle is considered as a release gap. Flexibility in using this filter is achieved by setting a special smoothing parameter.
Depending on the number of indicators used, the Hodrick-Prescott filter can be one-dimensional (when only output level data is used) and multidimensional (when other variables are taken into account). The main drawback of the one-dimensional Hodrick-Prescott filter is an exclusively statistical way of estimating potential output, without using additional information. This is also why there is an endpoint problem when the results are too sensitive to extreme observations (due to the inability to include past and future values). It is the multidimensional Hodrick-Prescott filter that allows us to take into account additional information and partially solve the problem of excessive sensitivity of extreme points.

Literature review

For the first time, a multidimensional filter was proposed by D. Laxton and R. Tetlow [1]. They used information on several macroeconomic indicators through

accounting for the Phillips and Oaken curves to estimate Canada's potential output. The applied method has led to better results than just a one-dimensional filter.

Among foreign articles, it is also worth noting the work of S. Borio and P. Disyatat [2], in which the output gap for the US economy was estimated in the period from 1980 to 2011 using a multidimensional Hodrick-Prescott filter. Borio and Disyat limited themselves to using only the Phillips curve.

Domestic researchers - A. Zubarev, P. Trunin [3; 4] - evaluate the "output gap" indicator for the Russian economy in the period from 2000 to 2015 using single- and multidimensional variants of the Hodrick-Prescott filter (the Phillips curve is taken into account).

There are quite a few works among domestic and foreign literature where the multidimensional Hodrick-Prescott filter is used on real data. And domestic works where both the Phillips curve and the Oaken law would be used at the same time were not found at all.

Data and research methods

To take into account information about additional indicators, the standard optimization problem, compiled with a one-dimensional Hodrick-Prescott filter, includes constraints based on assumptions about the structural relationships between different indicators.

The main idea of the multidimensional Hodrick-Prescott filter is to determine the potential output by minimizing the weighted average deviation from such values, changes in growth rates and errors in structural relationships.

The simplest modification of the one-dimensional Hodrick-Prescott filter, when the Phillips curve is used to estimate the potential output, can be represented as follows:

$$\min \sum_{t=1}^{T} (y_t - \tau_t^y)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1}^y - \tau_t^y) - (\tau_t^y - \tau_{t-1}^y)]^2 + \lambda^{\pi} \sum_{t=1}^{T} (e_t^{\pi})^2 ,$$

where y_t - the actual output, τ_t^y - the potential output, T – the sample size, e_t^{π} - the residuals of the Phillips curve equation, λ – the degree of smoothness of the series (for monthly data, it is better to take λ = 14400) [3], λ^{π} - the error smoothing

parameter (it is better to take $\lambda^{\pi} = 20$) [2].

The Phillips curve equation itself is better to take a hybrid type (both back-looking and forward-looking):

$$\pi_t = c + \beta_1 \pi_{t-1} + \beta_2 \pi_{t+1} + \beta_3 z_t + e_t^{\pi},$$

where $z_t = y_t - \tau_t^y$ - the output gap, π_t – inflation, e_t^{π} - the residuals obtained after evaluating the Phillips curve equation.

An iterative procedure is used to evaluate the potential release by a multidimensional Hodrick-Prescott filter:

1) evaluation of a potential release based on a one-dimensional HP filter;

2) inclusion of the obtained estimate as an independent variable in an econometric model corresponding to additional components (in this case, in the equation of the Phillips curve);

3) using the errors obtained from the Phillips curve in the objective function of the multidimensional HP filter for further evaluation of the "updated" potential release.

Steps 2 and 3 of the described procedure are repeated one after another until the convergence of the output gap estimate to some finite value is achieved.

To estimate the potential output using a multidimensional Hodrick-Prescott filter, not one, but, as proposed by Laxton and Tetlow, several structural equations can be used. Then the corresponding optimization problem looks like this:

$$\min \sum_{t=1}^{T} (y_t - \tau_t^y)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1}^y - \tau_t^y) - (\tau_t^y - \tau_{t-1}^y)]^2 + \lambda^{\pi} \sum_{t=1}^{T} (e_t^{\pi})^2 + \lambda^{u} \sum_{t=1}^{T} (e_t^{u})^2$$

The equation of Oaken 's law is better evaluated as follows:

$$u_t - u_{t-1} = a + \beta z_t + e_t^u$$

where u_t - the unemployment rate in %, e_t^u - the residuals of the equation of the Oaken curve, λ^u - the error smoothing parameter (selected independently).

A simple OLS is sufficient to evaluate the Phillips and Oaken equations, and an ARIMA model will be used for forecasting. The statistical package R and Excel will be used for all calculations and forecasts.

The output is the index of the physical volume of the GRP of the Siberian

District, in % compared to the previous month. At the same time, GRP must be evaluated independently – the weighted sum of output in several industries serves as an estimate of the total output: industry, construction, retail trade, provision of services. To construct and evaluate the Phillips and Oaken curves, the series of inflation by district (in%) and unemployment (in%) are additionally taken. All data for the period from January 2011 to May 2021 were uploaded from the Rosstat website.

Results

As mentioned earlier, the first step is to evaluate the potential release and rupture based on a one-dimensional Hodrick-Prescott filter. To apply a one-dimensional HP filter, R has a built-in package "mFilter", with the option "filter="HP"". The resulting output gap series is then used as an independent variable in the equations of the Phillips and Oaken curve.

The error series obtained after evaluating the two above equations are used for the multidimensional Hodrick-Prescott filter. After two or three repetitions of similar procedures, the series of potential release and rupture almost cease to change, so we can assume that the convergence of the series estimates to some finite value has been achieved.

To build a forecast (6 steps ahead), a function built into R was used that selects the ARIMA model that best describes the series, corresponding to the minimum AIC criterion. All models selected in R have the p-value of the Ljung-Box test above 0.05. This means that there is white noise in the residuals, i.e. there is no correlation between the residuals. And this is a sign of a good forecast.

The graphs below demonstrate the series of potential GRP and output gap and their forecasts obtained by the one-dimensional and multidimensional HP filter in two variations:

1) taking into account only the Phillips curve;

2) with simultaneous consideration of both the Phillips curve and the Oaken curve.

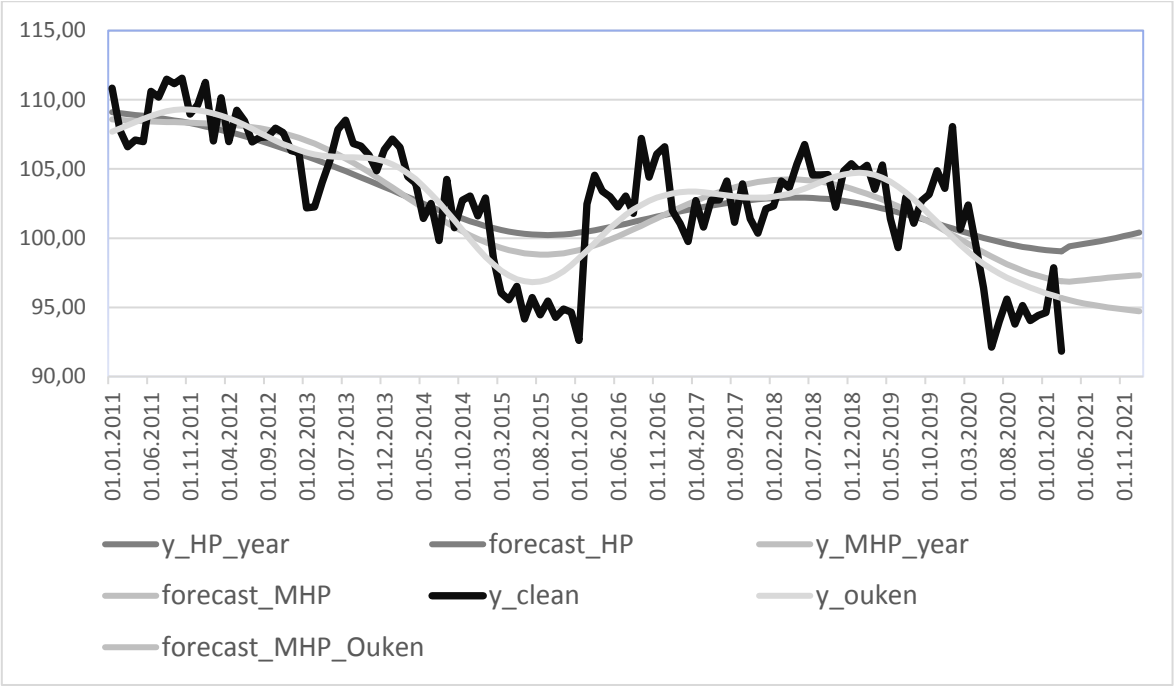

Figure 1 - Potential issue of the Siberian District

It follows from Figure 1 that the potential output level obtained using the HP multidimensional filter more accurately describes the dynamics of the actual output series. The output level (including potential), based on the forecasts obtained using the multidimensional HP filter only taking into account the Phillips curve, will grow, but more smoothly than based on the forecasts obtained using the one-dimensional HP filter.

The potential output level obtained using a multidimensional HP filter that takes into account both the Phillips curve and the Oaken curve describes the dynamics of the actual output series even more accurately. Moreover, the forecast obtained in this way, unlike other methods, indicates a continued decline in the output level.

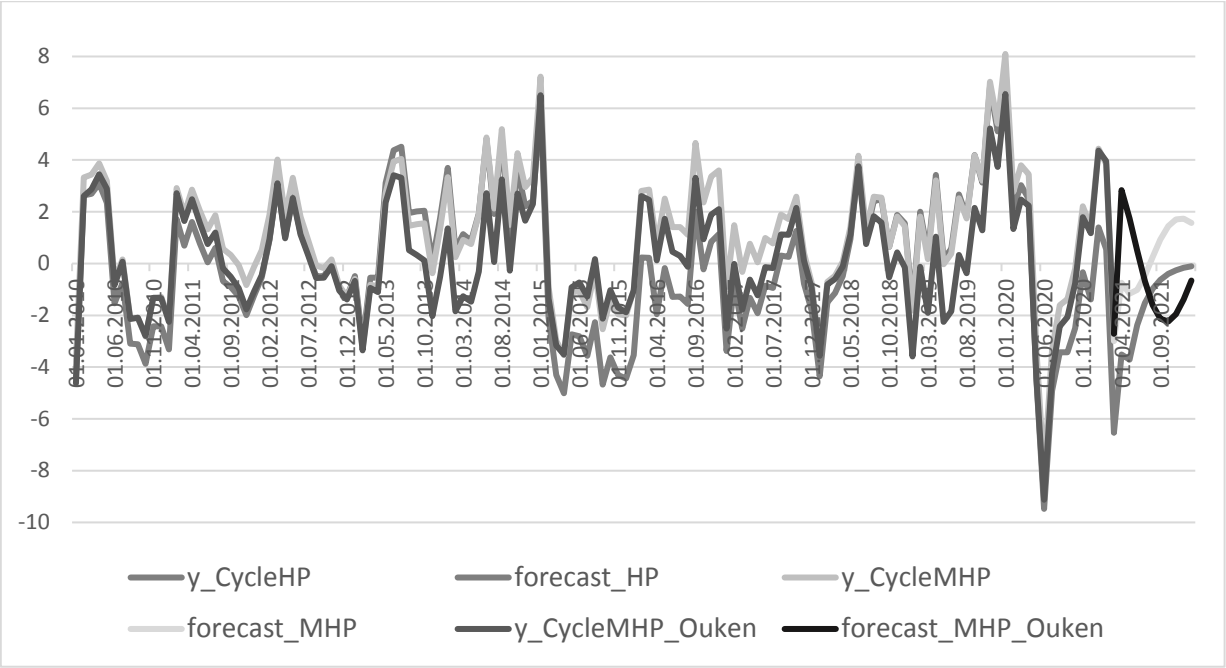

Figure 2 - Output gap of the Siberian District

Now let's analyze Figure 2. According to the calculations of the Central Bank, quarterly inflation in 2021 and 2022 will be positive, and it will peak during this period in June-September 2021. The results obtained with the help of the multidimensional HP filter indicate the same.

The forecast obtained using a multidimensional HP filter that takes into account both the Phillips curve and the Oaken curve is the opposite of forecasts obtained by other methods. Also, such a filter, taking into account both curves, made it possible to identify a problem in statistical data – overestimated unemployment rates during the coronavirus period. That is, such 2020 statistics do not accurately convey the economic reality. In this case, it is necessary, firstly, to revise the statistics, and secondly, to remove the release occurring at the time of the coronavirus 2020 from the data when using them.

The general conclusions that the study allows us to draw are as follows:

1. A multidimensional Hodrick-Prescott filter leads to more accurate results than a one-dimensional filter. Moreover, to assess and forecast the output gap, it is better to use both the Phillips curve and the Oaken law at the same time.

2. Statistics on the unemployment rate during the coronavirus period should be

reviewed.

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